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TITLE: Operation on chart, improvements

PERIODICAL: Shu Hsüeh Hsüeh Pao, v. 10, 3, 1960, 267-275

TEXT: An improved process for operation on charts is proposed by which an optimal flow chart is determined by checking a few circles only. A chart is interpreted to be a continuous linear chart on a plane. A basic flow chart is defined as one which satisfies the following conditions: 1) no opposite flow; 2) no circle having flow on all arcs; 3) number of arcs having flow less than number of points by one. A point P on a continuous linear chart is said to be a movable point, if the chart is still continuous after P and all arcs having P as an end point are deleted from the chart. The following are theorems concerning a basic flow chart: Theorem 1. There exists a basic flow chart. Theorem 2. Optimal flow chart may definitely be found from the basic flow charts. Theorem 3. Suppose that a basic flow chart is given.

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Then, for every arc having no flow (empty arc), there exists the only circle which includes the given empty arc and on which all other arcs have flows. Theorem 4. A basic flow chart is optimal if length of any flow on the outside and inside circles corresponding to all empty arcs in Theorem 3 does not exceed one-half circle length. In proving these theorems, the problems to be solved are expressed by mathematical expressions. Let C_{ij} denote distance between P_i, P_j; c_{ij}, length of arc connecting P_i, P_j. When there is no arc connecting P_i, P_j, let c_{ij} be a large positive integer M such that direct transportation between P_i, P_j is impossible. Let x_{ij} be the transportation volume from P_i to P_j, then for every point P_i, we have

$$\sum_{\substack{k=1\\k\neq i}}^{n} x_{ik} - \sum_{\substack{k=1\\k\neq i}}^{n} x_{ki} = a_i, \qquad i = 1, \cdots, n \qquad (1)$$

$$\left(\sum_{i=1}^{n} a_i = 0\right)$$

i.e., total issuing volume of P_i should be equal to the negative number of issuing or

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receiving volume of P_i. But our purpose is to find a set of non-negative x_{ij} ($i, j = 1, \ldots, n$, $i \neq j$) which satisfies (1) and such that the function

$$S = \sum_{\substack{i,j=1\\i\neq j}}^{n} c_{ij} x_{ij} \qquad (2)$$

has a minimum value. This is a linear programming problem and that determined by (1) and (2) is called Type A problem. By arranging variables of (1) as $x_{12}, \ldots, x_{1n}; x_{21}, \ldots, x_{2n}, \ldots; x_{n1}, \ldots, x_{n\,n-1}$, augmented matrix of (1) will have the following form:

$$B = \left[\begin{array}{cccccccccccccc} 1 & 1 & \cdots & 1 & -1 & & & & & & -1 & & & a_1 \\ -1 & & & & 1 & 1 & \cdots & 1 & \cdots & & & -1 & & a_2 \\ & -1 & & & & -1 & & & \cdots & & & & & \vdots \\ & & \ddots & & & & \ddots & & \cdots & & & & -1 & a_{n-1} \\ & & & -1 & & & & -1 & & 1 & 1 & \cdots & 1 & a_n \end{array}\right] \qquad (3)$$

Obviously, B has the rank n - 1 and thus the basis of (1) should comprise n - 1 vari-

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ables. Let x_{ij} be a variable of (1). Transformation of x_{ij} to x_{ji} is called inversion. Let

$$x_{i_1 j_1},\ x_{i_2 j_2},\ \ldots,\ x_{i_t j_t} \qquad (4)$$

be the t ($t \geqslant 2$) different variables of (1). If (4) itself or the variable set derived after inversion of certain variables in (4) can be arranged in the form

$$x_{k_1 k_2},\ x_{k_2 k_3},\ \ldots,\ x_{k_t k_1}, \qquad (5)$$

$$(k_1,\ k_2,\ \ldots,\ k_n \text{ being different})$$

then, set (4) is called a return route. The total of variables not inverted in arranging set (4) into form (5) is said to be the regular return route; the total of those remaining, the reverse return route. Lemma 4. The necessary and sufficient condition for coefficient row vectors

$$\alpha_{i_1 j_1},\ \alpha_{i_2 j_2},\ \ldots,\ \alpha_{i_t j_t} \qquad (6)$$

of (4) to be linearly dependent is that a portion of variables in (4) forms a return route.

Theorem 4'. A basic feasible solution set is given. If the length of either the regular or reverse return routes of each pair of nonbasic variables x_{ij} and x_{ji} does not exceed one-half the return route length, the given solution is the optimal solution. Theorem 4 is derived from Theorem 4' and determines the number of circles to be checked in checking a basic flow chart. Suppose that there are n points and s arcs in a given chart. In the basic flow chart, there will be n - 1 arcs having flows. Hence

Circle number to be checked = empty arc number = x - n + 1.

There are 8 figures. One English-language reference reads as follows: S. Vajda, Theory of game and linear programming.

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